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The scandal of deduction*

Hintikka on the information yield of deductive inferences

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Abstract

This article provides the first comprehensive reconstruction and analysis of Hintikka's attempt to obtain a measure of the information yield of deductive inferences. The reconstruction is detailed by necessity due to the originality of Hintikka's contribution. The analysis will turn out to be destructive. It dismisses Hintikka's distinction between *surface information* and *depth information* as being of any utility towards obtaining a measure of the information yield of deductive inferences. Hintikka is right to identify the failure of canonical information theory to give an account of the information yield of deductions as a scandal, however this article demonstrates that his attempt to provide such an account fails. It fails primarily because it applies to only a restricted set of deductions in the polyadic predicate calculus, and fails to apply at all to the deductions in the monadic predicate calculus and the propositional calculus. Some corollaries of these facts are a number of undesirable and counterintuitive results concerning the proposed relation of linguistic meaning (and hence synonymy) with surface information. Some of these results will be seen to contradict Hintikka's stated aims, whilst others are seen to be false. The consequence is that the problem of obtaining a measure of the information yield of deductive inferences remains an open one. The failure of Hintikka's proposal will suggest that a purely syntactic approach to the problem be abandoned in favour of an intrinsically semantic one.

Keywords: semantic information, deductive infernce, logical truth, logical equivalence, constituents, distributive normal forms, attributive constituents, depth, depth probability, surface probability, depth information, surface information

1 Introduction

What do we learn when we do logic? The answer given by the canonical theory of classical semantic information (*CSI*) is that we learn nothing, or at least nothing objective beyond an agent-relative "surprise-value" or some such. In classical logic, for any collection of premises $P_1,\ldots,P_n$ and conclusion C such that $P_1,\ldots,P_n \vdash C$, the resulting deduction may be rewritten as $P_1 \wedge\ldots\wedge P_n \rightarrow C$ where this sentence is logically true. This is simply the *deduction theorem* for classical logic, and does not hold for other logics such as many-valued logics and substructural logics. *CSI* assigns to logical truths a zero measure of semantic information. This result is counterintuitive. Hintikka calls the failure of *CSI* to provide an account of the information yield of deductive inferences *The Scandal of Deduction,* on a par with Hume's *Scandal of Induction*. It is this scandal that Hintikka has attempted to redress with his distinction between *surface information* and *depth information*.

Hintikka takes the basic notion of *CSI* as developed by Bar-Hillel and Carnap and extends it to apply to a full polyadic first-order language, the polyadic predicate calculus. *CSI* does not apply beyond a monadic first-order language. Hintikka uses 'tautological' and 'analytical' to denote deductive inferences (1973a, 222). This is due to the fact expounded above, that a conditional that takes as its antecedent the conjunction of a deductive inference's premises and as its consequent the inference's conclusion is logically true. If deductive reasoning is "tautological", and if logical truths have no "empirical content" and cannot make "factual assertions" then in what sense, Hintikka asks, does deductive reasoning give us new information (*ibid*)? Hintikka does not contest the *a priori* status of logical truths. He questions how it is that deductive inference (and mathematics, although this is only mentioned and then left aside) may impart new information to us. By 'logical inference' Hintikka means a *formal deductive inference* of elementary logic (the propositional and predicate calculi). His dispute is with the positivist position exemplified by Bar-Hillel and Carnap and explicated in full at the end of the present section, namely, that the information given in deductive inferences is of a purely psychological character.

CSI is built upon the classical modal space that Carnap (1955, 1956) used to define his notion of *intension* and which is commonly used to explicate metaphysical necessity.[1] The

[1] Bar-Hillel and Carnap built *CSI* around a monadic predicate language. The number of possible worlds is calculated accordingly. Where there are n individual constants (standing for n individuals) and m primitive monadic predicates, the number of atomic sentences will be nm, the number of possible worlds 2^{nm}, and the number of "Q-predicators" 2^m (Q-predicators are individuations of possible types of objects given a conjunction of predicates whereby each primitive predicate occurs

intension of a declarative sentence is taken to be the set of possible worlds that make the sentence true (equivalently, those worlds included by the sentence).[2] The notion of intension is co-definable with Bar-Hillel and Carnap's notion of *semantic information* as comprised by *CSI*. Semantic information is also referred to as *content* and denoted by 'Cont'. The content of a declarative sentence is taken to be the set of possible worlds that make the sentence false (equivalently, those worlds excluded by the sentence). Letting W be the set of all possible worlds, and X be the set of possible worlds identified with the intension of a declarative sentence s, and Y be the set of possible worlds identified with the content of s, we have (1.1):

(1.1) $W \setminus X = Y$ iff $W \setminus Y = X$

Hence X and Y will always be mutually exclusive and jointly exhaustive on W (i.e., they are a partition on W). Explicitly, the content of s will be identical to the set of possible worlds included by the negation of s. This is just to say that content of s is identified with the set of possible worlds included by $\neg s$. Where $X \subseteq W$ we have (1.2):[3]

(1.2) $\text{Cont}(s) =_{df} \{x \in W: x \vDash \neg s\}$

For any logically true sentence t, $\neg t$ will exclude every possible world. Via (1.2) we have (1.3):

(1.3) $\text{Cont}(t) = \varnothing$

CSI is concerned not only with the individuation of semantic information (Cont) but also with its *measure*. The guiding intuition is that the informativeness of a sentence s is inversely proportionate to the probability of the state of affairs it describes being the case. *CSI* involves two distinct methods for obtaining measures of semantic information, a *content measure* (cont) and an *information measure* (inf).

Beginning with cont, Bar-Hillel and Carnap denote the logical (*a priori*) probability of a sentence s by $m(s)$, where m designates 'measure' (*op. cit.*, 302). M is acquired via an *a*

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either negated or un-negated (but not both)). A full sentence of a Q-predicator is a Q-sentence where a predicate is attached to a term. Hence a possible world is a conjunction of n Q sentences as each Q-sentence describes a possibly existing individual.

[2] We speak of the intension associated with a declarative sentence as opposed to the intension associated with a *proposition* because, on the possible worlds understanding of propositions, a proposition *just is* an intension.

[3] We require '$\subseteq$' instead of the stronger '$\subset$' here because of the possibility that $\vDash \neg s$, in which case $X = W$.

priori probability distribution onto the set of all possible worlds. The distributed values sum to 1. For simplicity's sake, we may assume that the distribution pattern is equiprobable.[4] *CSI* defines the cont of a sentence s as the measure of the complement of s, (1.4):

(1.4) $\text{cont}(s) =_{df} 1 - m(s)$

A logically true sentence t is true in every possible world, hence (1.5):

(1.5) $m(t) = 1$

A logically true sentence will return a minimal content measure. From (1.4) and (1.5) we have (1.6):

(1.6) $\text{cont}(t) = 1 - 1 = 0$

Bar-Hillel and Carnap introduced the notion of an *information measure* (inf) to capture additivity on *inductive independence*. Two sentences are said to be inductively independent when the conditional probability of each sentence given the other is identical to its initial probability. Additivity on inductive independence fails for cont. For any two arbitrary sentences s and s', we cannot guarantee that $\text{cont}(s \wedge s') = \text{cont}(s) + \text{cont}(s')$ because it may be the case that $m(s)$ and $m(s)'$ have worlds in common. s and s' may have shared *content*. For additivity to hold on cont, it is *content independence* (not inductive independence) that is required.

The definition of inf may proceed via either cont (1.7) or m (1.8):

(1.7) $\text{inf}(s) =_{df} \log_2 \dfrac{1}{1 - \text{cont}(s)}$

(1.8) $\text{inf}(s) =_{df} \log_2 \dfrac{1}{m(s)} = -\log_2 m(s)$

(1.7) and (1.8) are equivalent, hence we consider only (1.7). Similarly to cont, any logically true sentence t will return a minimal information measure. From (1.7) and (1.6) we have (1.9):

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4 As we are only considering logically equivalent sentences, this assumption is (strictly speaking) irrelevant for present purposes.

(1.9) $$\mathrm{inf}(t) = \log_2 \frac{1}{1-0} = \log_2 \frac{1}{1} = \log_2 1 = 0$$

With respect to logically true sentences returning a zero value, the authors of *CSI* comment that:

> This, however, is by no means to be understood as implying that there is no good sense of 'amount of information' in which the amount of information of these sentences will not be zero at all, and for some people, might even be rather high. To avoid ambiguities, we shall use the adjective 'semantic' to differentiate both the presystematic sense of 'information' in which we are interested at the moment and their systematic explicata from other senses (such as "amount of psychological information for the person P") and their explicata (*ibid*, 223).

According to Hintikka, to assent to this positivist position would be to assent to the claim that "philosophical activity is a species of brainwashing" (*ibid*, 223). Hintikka notes that positivists have also claimed that the information carried by inferences concerns our conceptual systems or our linguistic usage (*ibid*, fn.5). Although he will concede something to this claim, he will do so in a manner that takes such information to be susceptible to an *objective measure*. Hintikka wants to construct a notion of quantifiable information given in deductive inferences. He uses 'psychological' to mean 'subjective', hence 'non-psychological conceptual facts' is not an oxymoron. Hintikka will claim that these non-psychological conceptual facts are the concern (in at least some sense) of the information involved in deductive inferences (and logical truths).

We will see how Hintikka's wider proposal is misleading and ultimately unfulfilled. The non-zero information measure he develops applies only to certain deductions of the polyadic predicate calculus. Certain other deductions of the polyadic predicate calculus will still receive a zero information measure even in Hintikka's adjusted sense. In addition to this, all of the deductions of a first-order monadic language as well as all of the deductions in the propositional calculus will receive a similar zero information measure. This is just to say that by Hintikka's own theory, only a proper subset of deductive inferences (indeed, a proper subset of those inferences in the polyadic predicate calculus) will count as non-psychologically informative. We note that Bar-Hillel and Carnap's positivist position on semantic information was formulated only for a monadic first-order language. We may presume some of their conclusions of a theory of semantic information to generalise to the simpler case of the propositional calculus. However, we may not presume that its conclusions will generalise to the more complex case of a full polyadic first-order language (the polyadic predicate calculus). It may be the case that Bar-Hillel and Carnap most likely hoped that their theory of semantic information was extendable to the full polyadic first-order case. In this sense Hintikka can quite rightly say that he has improved their theory by constructing such an extension. However, Hintikka's new notion of a non-zero information measure for deductive inferences applies *only* to the polyadic case (and then only partially at that). As such, Hintikka's proposal (partially) applies to a formal language outside the terms of reference of the theory of semantic information he criticises. His proposal fails to apply at all to the majority of deductive cases. Hence it fails to counter the positivist position on psychological information that Hintikka promises to refute.

Making this clear requires an explication of Hintikka's notions of *constituent*, *distributive normal form*, and *depth*. These will then be used to explicate Hintikka's notions of *trivially* and *non-trivially inconsistent constituents*. Then these latter notions will be used, via the notions of *depth probability* and *surface probability*, to expound Hintikka's notions of *depth information*, and *surface information*. The exposition that follows is unavoidably technical due to the nature of Hintikka's proposal.

2 Constituents and distributive normal forms for the propositional calculus and a monadic first-order language

Intuitively, *constituents* may be thought of as syntactically specific descriptions of possible worlds. Again intuitively, we may think of the *distributive normal form* of a sentence *s* as a syntactically specific description of the possible worlds that make *s* true. More specifically, but still intuitively, the distributive normal form of a sentence *s* is a disjunction of its constituents. We begin with the details for the propositional calculus case before moving into the details for the case involving a monadic first-order language.

A sentence in the propositional calculus will take as its constituents sentences corresponding to the rows in its truth-table that make it true. For example, $\neg p \vee q$ will take as its distributive normal form the disjunction of the sentences specified by rows 1, 3, and 4 of Table 1 below. A contradiction, or inconsistent sentence, will be the special case where the sentence's distributive normal form contains zero disjuncts. We stipulate such a formula to be $\perp$.

	p	q	$\neg p \vee q$
C_1	T	T	T
C_2	T	F	F
C_3	F	T	T
C_4	F	F	T

Table 1

The distributive normal form of $\neg p \vee q$ will be (2.1).

(2.1) $C_1 \vee C_2 \vee C_3$

When fully expanded (2.1) is written as (2.2).

(2.2) $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

A sentence's distributive normal form will quickly become typographically cumbersome, hence (2.2) is the last explicit representation we will deal with.

In the case of a monadic first-order language, specifying the distributive normal form of a sentence *s* in that language requires that we exhaustively specify, in a syntactically specific manner to be expounded presently, the consistent clusters of properties that possibly existing individuals may possess. Each of these consistent clusters is an *attributive constituent*. Intuitively speaking, in a monadic first-order language, attributive constituents are used to build up constituents in much the same way as constituents are used to build up distributive normal forms.

Formally, the type of sentence used to describe a constituent in the predicate calculus will have the following form (where $A_1, A_2, \ldots, A_k$ are unanalysed atomic formulas without free variables).

(2.3) $(\pm)\ A_1 \wedge (\pm)\ A_2 \wedge \ldots \wedge (\pm)\ A_k$

(2.3)'s exposition is contingent upon the richness of the predicate language under consideration. For the monadic case, specifying a constituent (that is, cashing out (2.3)) will involve specifying the possible types of individuals allowed via the expressive resources of the language. Here we have a number of one-place predicates $P_1, P_2, \ldots, P_k$, and an infinite number of bindable variables $x, y, z, \ldots$. This gives us expressions of the form $P_i(x)$. We also have the quantifiers (∀…), (∃…) and the usual logical connectives. In this case an individual will be specified as in (2.4).

(2.4) $(\pm)\, P_1(x) \wedge (\pm)\, P_2(x) \wedge \ldots \wedge (\pm)\, P_k(x)$

Each of the expressions of the type represented by (2.4) specifies a complex predicate and is an *attributive constituent* because it specifies the attributes possessed (and not possessed) by an individual assigned to x that satisfies it. Each '±' is a representation of the answer 'yes' or 'no' to the question of whether an individual assigned to x possesses the property denoted by 'P_i'. In other words, $(\pm)\, P_i(x)$ is either $P_i(x)$ or $\neg\, P_i(x)$ (but not both). These complex predicates are identical to Bar-Hillel and Carnap's *Q-predicators* (see fn. 1, §1 above) hence there will be $2^k = K$ attributive constituents where there are k distinct one-place predicates (Hintikka, 1970, 266). In what follows, we follow Hintikka's notation as closely as possible.

Hintikka (*op. cit.*) assumes that each of these Q-predicators or attributive constituents comes in some order so that we can enumerate them as in (2.5). '*Ct*' is used instead of '*C*' to distinguish attributive constituents from constituents proper, such as those in (2.1) above.

(2.5) $Ct_1(x), Ct_2, \ldots, Ct_K(x)$

Each formula $Ct_i(x)$ may be true of, and hence denote, a particular type of possible individual (that is, is an expression of the same type as (2.4)). These individuals may be used to specify possible worlds. This specification is accomplished by running through the list of attributive constituents in (2.5) and indicating whether or not the type of individual specified by each attributive constituent gets instantiated. In this manner, we *use attributive constituents* to specify the *constituents* of a monadic first-order language. These constituents have the form exhibited by (2.6).

(2.6) $(\pm)\, (\exists x)Ct_1(x) \wedge (\pm)\, (\exists x)Ct_2(x) \wedge \ldots \wedge (\pm)\, (\exists x)Ct_K(x)$

Considerations of simplicity cause Hintikka to stipulate a form for the constituents of the monadic first-order predicate language that differs from that given by (2.6). We will see how this is extended to the polyadic case in §3. Instead of running through the list of all the attributive constituents in (2.5) an indicating whether or not each attributive constituent is instantiated, we can list only those attributive constituents that get instantiated and then note that this list is complete. Where $\{Ct_{i_1}(x), Ct_{i_2}(x),\ldots, Ct_{i_w}(x)\}$ is a subset of the set of all attributive constituents listed in (2.5) we have (2.7).

(2.7) $$(\exists x)\, Ct_{i_1}(x) \wedge (\exists x)\, Ct_{i_2}(x) \wedge (\exists x) \wedge \ldots \wedge (\exists x)\, Ct_{i_w}(x)$$
$$\wedge\, (\forall x)[Ct_{i_1}(x) \vee Ct_{i_2}(x) \vee \ldots \vee Ct_{i_w}(x)]$$

For any sentence *s* of our monadic first-order language, the distributive normal form of *s* can be represented via a disjunction of constituents of the type exemplified by (2.7). Where $\{C_{i_1}, C_{i_2},\ldots, C_{i_{w(s)}}\}$ is a subset of the set of all the constituents of this type, we have (2.8).

(2.8) $$\text{dnf}(s) = C_{i_1} \vee C_{i_2} \vee \ldots \vee C_{i_{w(s)}}$$

That is, the distributive normal form of a sentence *s* is a disjunction of those constituents that entail *s*. The constituents not included in the distributive normal form of *s* are those constituents that are inconsistent with *s*. *s* will be logically equivalent to its distributive normal form. As with the case for the propositional calculus in (2.1) above, if *s* is a contradiction its distributive normal form will contain zero disjuncts. What sort of formula would this be? A disjunction with no disjuncts is difficult to write down. Hintikka does not say how he intends this to be accomplished, however we stipulate again that it is the formula $\bot$. The set $\{i_1, i_2,\ldots, i_{w(s)}\}$ is the *index set* of *s* and it is denoted by *I*(*s*). The index set *s* specifies the relevant set of constituents by indicating their position in the set of all constituents. The set of all constituents is ordered, given the assumption of ordering on their attributive constituents (see the paragraph preceding (2.5) above). The number *w*(*s*) is the *width* of *s*. The width of *s* indicates the number of constituents that support the truth of *s* (i.e., that entail *s*). It indicates how widely *s* spans across the set of all constituents.

Identically to Bar-Hillel and Carnap's method of assigning a logical (*a priori*) probability distribution across the space of all possible worlds that sums to 1, Hintikka assigns a probability distribution across the space of all constituents that sums to 1. Letting '*p*' denote the logical probability or *weight* (identical to Bar-Hillel and Carnap's use of '*m*', see §1 above) we can see that the logical probability of *s* is the sum of all the weights of its associated constituents. By definition these are the indices of the members of *I*(*s*). Hence we have (2.9).

(2.9) $$p(s) = \sum_{i \in I(s)} p(C_i)$$

From (2.9) Hintikka obtains measures of semantic information via 'cont' (1.4) and 'inf' (1.7) in an identical manner to that that of *CSI*. Bar-Hillel and Carnap do not consider first-order languages of greater complexity than a monadic first-order language. With a full polyadic language the issues are considerably more difficult.

3 The polyadic predicate calculus and depth

There is no general method for calculating the information of sentences in a full polyadic first-order language, that is, the polyadic predicate calculus. To do so requires making a finite list of the descriptions of the basic possibilities (i.e., constituents). Given the undecidability of the predicate calculus this cannot be done without restrictions being placed upon the expressive capabilities of the language. This failure of the notion of semantic information as it stands to be more than a methodological principle (as opposed to an actually executable procedure) with respect to the polyadic predicate calculus is the motivation behind Hintikka's formulating his notions of *depth information* and *surface information*. Explicating depth information and surface information turns on the details of formulating constituents for the predicate calculus. Explicating the formulation of constituents for the polyadic predicate calculus turns on the details of the aforementioned restrictions on the expressive capabilities of the language. This latter point turns on the notion of *depth*.

An important relationship not yet mentioned exists between (2.5) and (2.7). All points made in this paragraph with respect to (2.7) may be made *mutatis mutandis* for (2.6). The constituent in (2.7) contains only one *level* of quantification. This is despite (2.7) containing many quantifiers. For there to be more than one level of quantification some quantifiers must occur within the scope of others. This is not the case for (2.7). The number of levels of quantification occurring in an expression is that expression's *depth*, hence (2.7) is of depth $d = 1$.[5] (2.7) is defined out of attributive constituents (descriptions of possibly existing individuals) of the sort exemplified by (2.5). The attributive constituents in (2.5) are quantifier-free, that is, of depth $d - 1 = 0$. Hintikka appeals to this fact in order to give a recursive definition of attributive constituents and constituents in general (i.e., for both the monadic and polyadic case). For any constituent of depth d, it is composed out of attributive constituents of depth $d - 1$. These will be defined in turn out of attributive constituents of

[5] There is a *very* slight technical qualification here concerning the definition of depth, however, it is of no bearing whatsoever on our discussion. The curious reader is directed to (*ibid*, fn. 10, 295-6).

depth $d - 1 - 1 = d - 2$, decreasing depth one layer of quantification at a time until we "bottom out" at $d = 0$. Equivalently, by adding layers of quantification (as was done with (2.5) to generate (2.7)) we increase the level of depth. This move from (2.5) to (2.7) is the first step in the recursive process.

A restriction on the number of quantifier layers, that is, a restriction on depth, is the restriction required for the construction of constituents for the polyadic predicate calculus, and hence the restriction that allows the calculation of surface information. The notion of a constituent is now (for the polyadic predicate calculus) relative to depth. This restriction makes the situation with the polyadic predicate calculus similar to the situation with the monadic first-order language above. Generalising, each sentence *s* of depth *d* (or less) can be transformed into a disjunction of constituents of depth *d* (and with the same predicates as *s*). In this case, each general sentence *s* of depth *d* will have (in analogy with (2.8) above) the distributive normal form specified by (3.1) for some subset $\{C_{i_1}^{(d)}, C_{i_2}^{(d)}, \ldots, C_{i_{w(s)}}^{(d)}\}$ of the set of all constituents of depth *d* and with appropriate predicates.

(3.1) $\quad \text{dnf}(s) = C_{i_1}^{(d)} \vee C_{i_2}^{(d)} \vee \ldots \vee C_{i_{w(s)}}^{(d)}$

Each constituent $C_i^{(d)}$ of depth *d* (superscripts being used to denote depth) may be expressed as a disjunction of constituents of depth $d + e$, which Hintikka calls the *expansion* of $C_i^{(d)}$ at depth $d + e$. Any constituent occurring within an expansion of $C_i^{(d)}$ is said to be *subordinate* to $C_i^{(d)}$ (*ibid*, 268). Clarifying the relevant details requires further explicating the notion of a constituent for the polyadic predicate calculus. Hintikka proceeds via a generalisation of the process outlined above with respect to a monadic first-order language. He defines the expressions that specify possibly existing objects, that is, attributive constituents, for the polyadic predicate calculus.

4 Attributive constituents for the polyadic predicate calculus

To begin this process, Hintikka first assumes a finite set of predicates. As noted above, the process of defining attributive constituents for the predicate calculus is recursive. We define a list of all the types of individuals *x* that are specifiable given *d* layers of quantifiers and fixed reference-point individuals named by $a_1, a_2, \ldots, a_m$. The expressions that specify these possible objects, that is, the attributive constituents of the predicate calculus, are as (4.1).

(4.1) $Ct_i^{(d)}(a_1, a_2, \ldots, a_m; x)$

(4.1) is the polyadic analogue to (2.4) above. It specifies that x is related to $a_1, a_2, \ldots, a_m$ by the ith attributive constituent with d layers of quantifiers (and appropriate predicates). As with the monadic case above, we can move through the list of attributive constituents such as (4.1) and indicate which members of the list are instantiated. This will give us the polyadic analogue to (2.6), (4.2). (4.2) is an attributive constituent of the polyadic predicate calculus at depth d (keep in mind that for full first-order polyadic languages, the notion of a constituent is now relativised to depth).

(4.2)
$$(\pm)(\exists x)\, Ct_1^{(d)}(a_1, a_2, \ldots, a_m; x)$$
$$\wedge (\pm)(\exists x)\, Ct_2^{(d)}(a_1, a_2, \ldots, a_m; x)$$
$$\wedge \ldots$$
$$\wedge (\pm) A_1(a_m) \wedge (\pm) A_2(a_m) \wedge \ldots$$

$A_1(a_m)$, $A_2(a_m)$,… specify all the atomic sentences that can be constructed from both the finite list of predicates and the constants $a_1, a_2, \ldots, a_m$, and which include at least one occurrence of a_m. The manner in which a_m is related to $a_1, a_2, \ldots, a_{m-1}$ can be specified via replacing the constant a_m with a bindable variable y, $d + 1$ layers of quantifiers and the correspondingly smaller set of reference-point individuals (i.e., those named by $a_1, a_2, \ldots, a_{m-1}$). Expressions such as (4.3) stand for expressions such as (4.4) because (4.4) contains an added layer of quantification, i.e., (4.3) = (4.4).

(4.3) $Ct_j^{(d+1)}(a_1, a_2, \ldots, a_{m-1}; y)$

(4.4)
$$(\pm)(\exists x)\ Ct_1^{(d)}(a_1, a_2, \ldots, a_{m-1}, y; x)$$
$$\wedge (\pm)(\exists x)\ Ct_2^{(d)}(a_1, a_2, \ldots, a_{m-1}, y; x)$$
$$\wedge \ldots$$
$$\wedge (\pm) A_1(y) \wedge (\pm) A_2(y) \wedge \ldots$$

Analogously to the manner by which (2.6) was rewritten as (2.7) above, (4.4) can be rewritten as (4.5).

$$
\begin{aligned}
(4.5)\qquad & (\exists x)\ Ct_1^{(d)}\ (a_1, a_2, \ldots, a_{m-1,}\ y; x)\\
& \wedge (\exists x)\ Ct_2^{(d)}\ (a_1, a_2, \ldots, a_{m-1,}\ y; x)\\
& \wedge \ldots\\
& \wedge (\forall x)[\ Ct_1^{(d)}\ (a_1, a_2, \ldots, a_{m-1,}\ y; x)\\
& \vee \qquad Ct_2^{(d)}\ (a_1, a_2, \ldots, a_{m-1,}\ y; x)\\
& \vee \qquad\qquad\qquad\qquad \ldots]\\
& \wedge (\pm)\ A_1\ (y) \wedge (\pm)\ A_2\ (y) \wedge \ldots
\end{aligned}
$$

When speaking of the attributive constituents of the predicate calculus we will be speaking of expressions of the form of (4.5) from now on. Each attributive constituent of depth $d + 1$ of the predicate calculus is defined in terms of attributive constituents of depth d. Each of these attributive constituents of depth d is defined in terms of attributive constituents of depth $d - 1$ etc., until, as noted in the second paragraph of §3 above, the process "bottoms out" at depth $d = 0$. With attributive constituents defined, we can now define constituents for the predicate calculus.

A constituent for the predicate calculus contains an exhaustive collection of attributive constituents (of the form of (4.5)). It will therefore contain *all* of the atomic statements specifiable by our set of predicates and finitely many constants, and not just those that contain at least one occurrence of a_m, or its replacement y as is the case with (4.5). Where $d = 0$, (4.5) will only consist of the bottom row, that is, (4.5) will be a constituent of the propositional calculus. In this case, we have the initial step of the aforementioned recursive definition of constituents. We now have a definition of constituents in general. On the basis of expressions such as (4.6), constituents of depth d are generated in an analogous manner to that via which the constituent (2.6) (then rewritten as (2.7)) was generated from the attributive constituents in (2.5) for the monadic case. The existential quantifier indicating instantiation will increase the depth by +1.

$$(4.6)\qquad Ct_j^{(d-1)}(x)$$

The explication of Hintikka's notions of depth, attributive constituent, and constituent for the polyadic predicate calculus is now complete.[6] The next task is to clarify Hintikka's notions of *trivially* and *non-trivially inconsistent constituents*. When this is done we will be in a position to understand his Completeness Theorem for distributive normal forms along

[6] For an exposition of Hintikka's notion of constituents in set-theoretic terms, see (Scott,1979).

with his notions of *depth probability* and *surface probability*. Finally, we will use depth probability and surface probability to elicit an understanding of Hintikka's distinction between *depth information* and *surface information*.

5 Trivially and non-trivially inconsistent constituents

The form of (4.5) ensures that all negation-signs have a "minimal scope" (1970, 271), which is to say that they contain neither connectives nor quantifiers within their scope. All of the other logical operations involved are *monotonic*. Taking conjunction as an example, this is to say that if $\alpha \vDash \beta$, $\alpha' \vDash \beta'$ then $\alpha \wedge \alpha' \vDash \beta \wedge \beta'$. Accordingly, any expression that can be generated via the omission of conjuncts in a constituent, or in an attributive constituent, is implied by the original expression. A layer of quantifiers in a constituent or an attributive constituent can be removed, and the expression generated will also be one that is implied by the original. This is due to the fact that bindable variables only enter into the expression via atomic expressions that consist of a predicate with bindable variables or constants, and that these atomic expressions only occur in conjunctions (where they may be negated). In this case, the generated expression will be like the original expression except that, in certain places, a constituent or attributive constituent of depth d will have been replaced by a conjunction of similar expressions of depth $d - 1$.

Distinct constituents and attributive constituents are incompatible by definition. Accordingly, certain constituents may be *inconsistent*. They may however, be inconsistent in two distinct ways. The first way in which a constituent may be inconsistent is by being *trivially inconsistent* (*ibid*, 272). Trivial inconsistency is a technical notion and Hintikka does not give any examples. Intuitively, it is commonplace that a pair of formulas may be inconsistent, yet certain syntactic manipulations must be carried out before the inconsistency is made explicit. This is a frequently occurring exercise in logic textbooks, especially when the proof method of *Reductio ad Absurdum* is being learned. Similarly, there is a decision procedure for trivial inconsistency at depth d. A constituent of depth d is said to be trivially inconsistent if its inconsistency can be seen at depth d via its syntax failing to satisfy certain syntactic rules. A constituent of depth d that must be expanded to some greater depth $d + e$ before its inconsistency is revealed in virtue of the trivial inconsistency of its subordinate constituents is said simply to be *inconsistent*. To avoid ambiguity on this point we will call inconsistent constituents *non-trivially inconsistent*. This point leads straight to Hintikka's *Completeness Theorem* of distributive normal forms.

6 Completeness, depth probability, surface probability, depth information, and surface information

In his (1965) and (1973b, ch. XI, §§15-17) Hintikka proves his Completeness Theorem for distributive normal forms: If $C_i^{(d)}$ is an inconsistent constituent (either trivially or non-trivially so) of the polyadic predicate calculus and $C_i^{(d)}$ is expanded into increasingly deeper distributive normal forms, there will be a depth $e \geq d$ such that every disjunct in the distributive normal form of $C_i^{(e)}$ is trivially inconsistent. Hence a constituent's inconsistency can always be made trivially obvious by increasing its depth. The complication here is that owing to the undecidability of the polyadic predicate calculus, we do not know how far a constituent will need to be expanded until its inconsistency is made transparent. As Hintikka puts it "this presence of potential inconsistencies is a fact we have to live with" (1970, 274).

Hintikka's notion of *depth probability* for a sentence *s* of the polyadic predicate calculus is calculated in much the same manner as is the probability for a sentence *s* in a monadic language expounded in (2.9) above. Here however, not only will the constituents involved be relativised to depth, but a positive probability weight will *only* be assigned to the consistent constituents of the polyadic predicate calculus. Where *s* is a sentence of the polyadic predicate calculus of depth ≤ *d* and, to borrow the more contemporary notation of Rantala and Tselishchev (1987)[7] dnf(*s*) = $\bigvee_{1 \leq i \leq n} C_i^{(d)}$ (i.e., the disjunction of constituents in dnf(*s*) is non-empty up to *n*), we have (6.1):

$$(6.1) \qquad p(s) = \sum_{i=1}^{n} p(C_i^{(d)})$$

To reiterate, each $C_i^{(d)}$ is consistent. Hence (6.1) involves the presupposition that, for every depth *n*, where *X* is the set of consistent constituents in the polyadic predicate calculus at depth *n*, the initial probability assignment sums to 1, that is that $p(\bigvee X) = 1$. In this case we have it that $p(s) = 1$ iff *s* is a logical truth, and $p(s) = 0$ iff *s* is a contradiction. Hence depth probability is used by Hintikka to define his notion of *depth information* via cont, that is $\text{cont}(s) =_{df} 1 - p(s)$ (note that this is identical, with *p* switched from *m*, to (1.4) in §1 above). 'inf' could be used just as well. Depth information will obviously have the familiar

[7] Rantla and Tselishchev provide what is perhaps the *only* (partial) reconstruction of Hintika's proposal in the literature. Their work has made this one immensely easier. For a detailed working out of the syntactic behaviour of constituents, see Rantala (1987). Bremer (2003) and Bremer & Cohnitz (2004, §2.3) also provide a brief (several pages) mention of Hintikka's proposal.

characteristics associated with cont from Bar-Hillel and Carnap's (*op. cit.*, §6), four of which we repeat here:[8]

(a) $\text{cont}(s) = 0$ iff $\vdash s$

(b) $\text{cont}(s) = 1$ iff $\vdash \neg s$

(c) $\text{cont}(s) > \text{cont}(s')$ if $\vdash s \supset s'$ but $\nvdash s' \supset s$

(d) $\text{cont}(s) = \text{cont}(s')$ if $\vdash s \equiv s'$

The depth information of s is not calculable in practice, however, as we cannot effectively isolate the inconsistent constituents of the predicate calculus. Being able to isolate the inconsistent constituents of the predicate calculus is equivalent to the predicate calculus being decidable, and it is not. As Rantala and Tselishchev (*op. cit.*, 82) make clear, this means that in general there is no decision procedure whereby the initial distribution of probability weights can get assigned, as the set of consistent constituents of the predicate calculus is not recursive. It is this fact about depth information that motivates Hintikka to construct his notion of *surface information*.

Hintikka defines surface information via the notion of *surface probability*. Surface probability is calculated via the distribution of a positive probability measure (a weight) to every constituent of the polyadic predicate calculus that is not *trivially inconsistent*. In this case both consistent and non-trivially inconsistent constituents will be assigned a weight. Rantala and Tselishchev clarify these issues with the assistance of several set-theoretic notions. Let s and s' stand for two sentences of the predicate calculus and stipulate that $s^{(d)} = \bigvee S$, and $s'^{(d)} = \bigvee S'$. In this case S and S' will be composed of constituents of depth d. They assume that $S \subset S'$ and that $X = S' \setminus S$ (the set of elements of S' that are not elements of S) contains only inconsistent constituents. In this case it is a fact that $p(s) = p(s')$ and hence that $\text{cont}(s) = \text{cont}\ (s')$. Their depth probability and hence the depth information contained by s and s' will be identical. The question is whether we may discern this fact simply by examining the syntax of $s^{(d)}$ and $s'^{(d)}$. If even one constituent in X is non-trivially inconsistent then this cannot be guaranteed. In this case, given the syntax of $s^{(d)}$ and $s'^{(d)}$ all that we can say is that s' *appears* to admit of more possibilities than does s, and hence appears to be more probable, and hence appears to contain less information in some sense of the term. As

[8] (a), (b) and (d) appear as (T6-4b), (T6-4c) and (T6-4f) on p. 238 of Bar-Hillel and Carnap's (*ibid*) respectively. (c) does not appear in its present form, however, it follows from (T6-4e-f) (*ibid*).

Rantala and Tselishchev put it, Hintikka intends his notion of surface information to capture and explicate “this kind of ‘apparent’ information” (*ibid*, 83).

We know from the completeness theorem of distributive normal forms that there will be a depth $e > d$ where all the constituents that are elements of X *can* be seen to be inconsistent, and this will be precisely when all of the subordinate constituents of depth e are *trivially* inconsistent. In the transition from depth d to depth e, the apparent possibilities (which are the elements of X) that cannot be made distinct from the genuine ones at depth d are revealed to be merely apparent possibilities at depth e. Surface information is the information type resulting from the elimination of such apparent possibilities.

The measure of depth probability assigned to a constituent $C^{(d)}$ gets distributed between all the consistent constituents subordinate to $C^{(d)}$ at depth $d + 1$. The situation with surface probability is different, as both consistent *and* non-trivially inconsistent constituents will receive a weight. Given that $C^{(d)}$ is non-trivially inconsistent, it is possible that each of its subordinate constituents at depth $d + 1$ are trivially inconsistent. In this situation, the weights are redistributed by Hintikka in the following manner (1973b, 229): assume that every constituent subordinate to $C^{(d)}$ at depth $d + 1$ *is* trivially inconsistent whilst $C^{(d)}$ is not (as stipulated above). Now assume that $C^{(e)}$ (where now $e < n$) is the first constituent *prior* to $C^{(d)}$ which has a non-trivially inconsistent subordinate constituent with a depth of $d + 1$. We know that there will be such constituents in virtue of the fact that $C^{(d)}$ is not trivially inconsistent. Taking X to be the set of all of these non-trivially inconsistent constituents of depth $d + 1$ that are subordinate to $C^{(e)}$, the probability weights will be distributed amongst the elements of X. The distribution ratio is decided in advance. The sum of the weights must be 1 at each depth and for simplicity’s sake we’ll assume an even distribution pattern. *Figure* 1 (slightly adapted from Rantala and Tselishchev, *op. cit.*) bears this out. The surface probability of $C^{(e)} = r$. White circles denote trivially inconsistent constituents and black circles denote consistent or non-trivially inconsistent constituents. The arrowed lines specify how it is that the weights are redistributed. The weight is divided equally among the subordinate constituents at the next level.

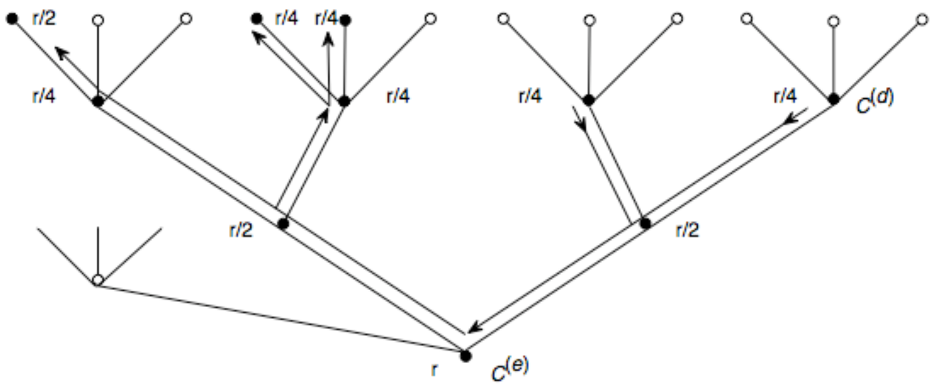

Figure 1

> *Figure* 1 demonstrates the redistribution of weights where that every constituent subordinate to $C^{(d)}$ at depth d + 1 is trivially inconsistent whilst $C^{(d)}$ is not, and where $C^{(e)}$ (where $e < n$) is the first constituent *prior* to $C^{(d)}$ which has a non-trivially inconsistent subordinate constituent with a depth of d + 1.

Hintikka uses p' to distinguish the measure of surface probability from that of depth probability p, hence in *figure* 1 above $p'(C^{(e)}) = r$. Once this weight ratio has been specified, there will be, at least in principle, a definite measure of surface probability assigned to each closed constituent (i.e., a constituent with no free variables) of the polyadic predicate calculus. He denotes the surface probability of a sentence s of the polyadic predicate calculus of depth ≥ d via $p^{(d)}(s)$. Hence, surface probability is relative to depth. Again borrowing the notation of Rantala and Tselishchev, we denote distributive normal form of s as $\bigvee_{1 \le i \le n} C_i^{(d)}$, in which case we have (6.2).

(6.2) $$p^{(d)}(s) = \sum_{i=1}^{n} p'(C_i^{(d)})$$

Hence the appropriate surface information measure denoted by $\text{cont}^{(d)}(s)$ is given by (6.3) (Hintikka, 1970, 285).

(6.3) $$\text{cont}^{(d)}(s) = 1 - p^{(d)}(s)$$

As Rantala and Tselishchev (*op. cit*., 85) note, (a)–(d) will fail for the measure of surface information. With regards to (a), a sentence s may be provable in the polyadic predicate calculus whilst cont$^{(d)}(s) > 0$ for some depth d. In particular, this will be the case where there is a non-trivially inconsistent constituent of the polyadic predicate calculus of depth d that does not occur in the distributive normal form of s at depth d. As a sentence is expanded into deeper distributive normal forms, its surface information may change accordingly (see Hintikka, 1970, 282-3). *Figures* 2 – 4 (again slightly adapted from Rantala and Tselishchev, *op. cit.*, 86) specify three distinct cases that demonstrate the different ways via which this may occur. The first case is where cont$^{(d+1)}(s) >$ cont$^{(d)}(s)$, the second case is where cont$^{(d+1)} =$ cont$^{(d)}(s)$, and the third where cont$^{(d+1)} <$ cont$^{(d)}(s)$.

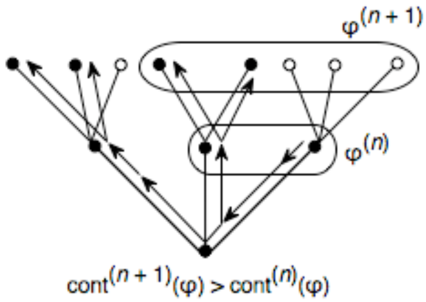

Figure 2

Figure 2 demonstrates how a sentence's expansion into deeper distributive normal forms causes a change in surface information, where cont$^{(d+1)}(s) >$ cont$^{(d)}(s)$.

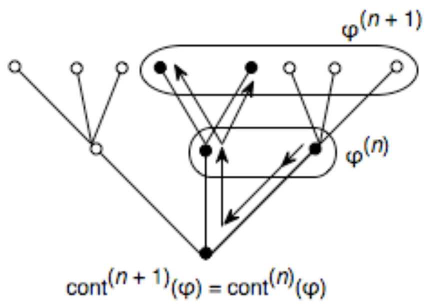

Figure 3

Figure 3 demonstrates how a sentence's expansion into deeper distributive normal forms causes a change in surface information, where cont$^{(d+1)} =$ cont$^{(d)}(s)$.

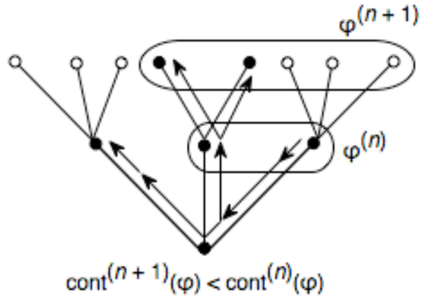

Figure 4

Figure 4 demonstrates how a sentence's expansion into deeper distributive normal forms causes a change in surface information, where $\text{cont}^{(d+1)} < \text{cont}^{(d)}(s)$.

Given that a general condition regarding weight distribution is adhered to however, the intuitive relation between surface information and depth information can be captured by (6.4). (6.4) states that depth information is the limit of surface information (Hintikka, *op. cit*, 283-4).

(6.4) $$\lim_{d \to \infty} \text{cont}^{(d)}(s) = \text{cont}(s)$$

The information yield of deductive inferences is defined in terms of the amount of surface information they yield. Hintikka accomplishes this via a general disproof procedure. Suppose that s is inconsistent (contradictory). Via the completeness theorem of distributive normal forms we know that there is a depth $e > d$ where $s^{(e)}$ will be the first expansion of $s^{(d)}$ such that every constituent will be trivially inconsistent (this is on the assumption that $e \neq d$ of course). Distributive normal forms can thus be used, at least in principle, for disproofs. There will be a change of surface information of s via its disproof. The surface information of s at depth e will be greater than its surface information at depth d, as $\text{cont}^{(e)}(s) = 1$ (as s is a contradiction). Specifically, the surface information yield is as follows (Hintikka, *ibid*, 285) (Rantala and Tselishchev, *op. cit*):

$$\begin{aligned} &\text{cont}^{(e)}(s) - \text{cont}^{(d)}(s) \\ &= 1 - \text{cont}^{(d)}(s) \\ &= p^{(d)}(s) \end{aligned}$$

Of course the surface information yield will be 0 where $e = d$. It will be non-zero for all other cases. Hintikka reduces other cases of deductive inferences to disproofs. Hence the proof of a *valid* sentence s will proceed via a *distributive normal form disproof* of $\neg s$.

7 Evaluation

We are now in a position to evaluate Hintikka's proposal. Although its ingenuity is unassailable, we must question whether it succeeds in its stated aim. This is to give a measure of the information yield of deductive inferences. Recalling that we can turn deductive inferences into logical truths via constructing a conditional that takes the conjunction of the inference's premises as its antecedent and the inference's conclusion as its consequent, we ask *has* Hintikka achieved the extraction of non-zero information measures of deductive inferences in particular and logical truths in general?

Hintikka has developed a new method of deductive inference. This is the method of disproof via distributive normal forms of the polyadic predicate calculus. A measure of surface information is thus only obtainable via this disproof method. As Rantala and Tselishchev remark, the method of disproof via distributive normal forms is "not very practical" in practice (*ibid*, 89, fn. 15). In this case, to what extent can Hintikka claim to having explicated the amount of semantic information generated by deductive inferences? The inferences that Hintikka's notion of surface information apparently hangs on are of an altogether different type to those that others (in particular the positivists whom Hintikka so derides) have claimed contain a zero-measure of semantic information. In this case, has Hintikka in any real sense *rebutted* the positivist's contention?

Answers to these questions depend on how strongly restricted we understand the connection between surface information on the one hand and the method of disproof via distributive normal forms on the other. Rantala and Tselishchev take the restriction to be so strong as to apply only to the proofs and sentences generated via Hintikka's disproof method. They write "So it seems that what is given to us by Hintikka by means of surface information is a way to measure…*only* the kind of information which is attainable by means of this specific method of proof" (*ibid*, 87, their italics). If this is right, then Hintikka's theory of surface and depth information is somewhat wide of its mark. We began with the issue of the information yield of deductive inferences for the propositional and predicate calculi. Presumably, we also began with the standard proof procedures, for example natural deduction or some axiomatic system was in mind. At any rate, some commonly practiced proof procedure was presumably taken for granted. If Rantala and Tselishchev are correct, then surface information only applies to sentences generated by Hintikka's disproof method.

In this case Hintikka's theory of surface and depth information *has* missed its target completely.

But are Rantala and Tselishchev correct? We should not be so fast. It is true that the required measures of surface information are only attainable via Hintikka's method of disproof. It is also true that the disproof procedure is not very practical. However, it is clear enough that Hintikka understands measures of surface information to apply to logically true and logically equivalent sentences *no matter what* proof procedure has been used in their derivation. The evidence for this is strong. Hintikka writes that:

> [A]ll the usual types of logical arguments – disproofs, proofs, proofs from premises, equivalence proofs – can in principle be carried out…by first converting the statements in question into their normal forms and then expanding them until at a great enough depth the desired logical relation [a contradiction] becomes apparent. In each case, there is a natural measure of surface information gained in the course of the argument (1970, 284-5).

More anecdotally, in the opening section of ch. X of his (1973b, 222-3), Hintikka frames his notion of surface information in terms of answering the question of how logical truths in general and deductive inferences in particular get to be informative. He uses an example of a "clever freshman" who asks his logic teacher of the informativeness of deductive inferences and logical truths. If Hintikka always had his method of disproof via distributive forms in mind, it is unlikely that he would use such an example. Hintikka's reference to the positivist position that the only sense in which deductive inferences and logical truths are informative is a non-objective psychological one is more evidence to this point. Clearly he is aware of the positivists *not* being aware of his extension to their theory of *CSI*.

With this being so, it is arguable that Hintikka understands his disproof method as an auxiliary process that allows us to identify and measure the informativeness of deductive inferences and logical truths irrespectively of the actual proof procedure used in their derivation. The complexity of Hintikka's disproof method apparently brings with it the exposure of the deep structure of deductions. If this is the case, Hintikka has not landed wide of the mark in the manner that Rantala and Tselishchev argue. However, Hintikka is not necessarily in a good position simply because of this. In fact, what good reasons do we have for believing that measures of surface information *do* capture some of the informativeness of deductive inferences and logical truths?

There is a sense in which Hintikka's notion of surface information is not something that anyone, positivists included, could be dissatisfied with as it *stands*. The dissatisfaction comes into play when we concentrate on the *claims* made about the notion of surface information by Hintikka on the one hand, and the *facts* about it on the other.

Surface information is supposed to give us a measure of the informativeness of deductive inferences. Even *granting* that the deductive inferences in question are restricted to disproofs via distributive normal forms, we note that an inference's surface information is calculated via the particular formulas involved in the inference in question. Distinguishing between the surface informational yield of distinct inferences proceeds via the addition of the information gained from the steps involved. Hence we could make one inference more informative than another simply by adding in irrelevant steps. Rantala and Tselishchev (*ibid*, 87-8) expound this fact and point out that for this proposal to be workable, Hintikka will need to add some qualifications. They note another technical shortcoming: inferences do not always proceed solely via means of sentences, but surface probability is only assigned to closed constituents. In order to account for the fact that inferences often proceed via formulas containing free variables, the notion of surface information will have to be generalised so as to also apply to constituents containing formulas with certain free variables. The difficulty concerns specifying the variables in question. One suggestion we're given is to take the set of all free variables involved in a particular deduction as those requiring specification for that deduction itself (*ibid*, 89, fn. 18).

However, even if we suppose that all such concerns can be circumvented, there is a much larger shortcoming of Hintikka's notion of surface information that is fatal for his proposal. Recall that Hintikka's proposal is, in his own words, "to answer the question: How does deductive reasoning add to our knowledge (information)" (1970, 289). He refers to the failure to answer this question as 'the scandal of deduction' (*ibid*). Has Hintikka really answered this question? We can see now that there is an important sense in which he has not. In fact, Hintikka's answer to this question only applies to the restricted set of deductive inferences that involve an increase in surface information. This set is more restricted than it might first appear. Hintikka refers to all inferences of the polyadic predicate calculus as *depth tautologies*, as they *may* all be seen as valid deductions via an increase in depth. It's important to keep in mind that we are working *within* Hintikka's system of proof-via-distributive-normal-forms here. Deductive inferences that do not necessarily require an increase in depth in order to be seen as valid are called *surface tautologies*. Let D be the set of depth tautologies and S the set of surface tautologies. We know that $S \subset D$. In this case only $\Delta = D \setminus S$ (the set of depth tautologies that are not also surface tautologies) will involve an increase in surface information. Hence only the deductions that are elements of Δ will 'add to our knowledge (information)'.

The obvious problem with this result is that *S* is large. Not only does it contain many polyadic deductions, it contains the entire set of deductive inferences of monadic first-order logic. This is due to the fact that no monadic constituents are inconsistent. Note also that all of the deductions in the propositional calculus will fail to generate any surface information. This is due to the fact that the propositional calculus contains only consistent constituents, and fails to contain quantifiers *simpliciter*. Since Hintikka formulated the notion of surface information to answer the question 'How does deductive reasoning add to our knowledge (information)?' his answer forces us to say that all propositional inferences, as well as all monadic inferences and certain polyadic ones, *fail* to add to our knowledge (information). This is due to the fact that they fail to generate an increase in surface information. Consider (6.5) and (6.6) from the propositional calculus and monadic predicate calculus respectively.

(6.5) $(A \lor B) \to (C \to D), (C \to (C \land D)) \to E, E \to ((\neg F \lor \neg\neg F) \to (A \land F))$
$\vdash A \leftrightarrow E$

(6.6) $(\forall x)(A(x) \to (\exists y)B(y)) \dashv\vdash (\exists y)((\exists x)A(x) \to B(y))$

Both (6.5) and (6.6) will receive a zero measure of surface information. This result is confusing in light of Hintikka's comment that "a great deal of ingenuity often goes into logical … argument. The result of such argument often enlightens us, gives us new insights, informs us greatly." (1970, 288). Hintikka's goal is to reject the position that "there is no objective sense in which a conclusion of a valid inference contains more information than its premises" (*ibid*, 289). Via his method of rejection, however, Hintikka is only able to assign an information-increase to the conclusion of an extremely limited collection of deductive inferences, namely a proper subset of inferences of the polyadic predicate calculus. What are left out are all the deductions in the propositional calculus, all of the deductions in a monadic first-order language, and certain inferences in the polyadic predicate calculus. However it seems that such deductions, despite not yielding any surface information, still "enlighten us, give us new insights, and inform us greatly".[9] Since Hintikka has proceeded in terms of an objective measure of the information yield of a *very* restricted subset of deductive inferences, we may still sensibly ask just what is it exactly that is being measured? What in short *is* surface information a measure of?

[9] At one point, Hintikka hints that he aware of this yet seems surprisingly unworried. In his reply to Rantala and Tselishchev, Hintikka writes that "My basic idea has been that an inference is uninformative if and only if it is corollarial in Peirce's sense, that is, does not need the aid of any auxiliary individuals (Bogdan, 1987, 282). We should be worried however, as Hintikka's claim here is clearly and obviously false (as it entails that the propositional calculus and monadic predicate calculus are entirely uninformative).

Hintikka notes that the following interpretation is tempting: Depth information is information about the objective reality that our statements are statements about, whilst surface information is information about the “conceptual system we use to deal with this reality (or aspect of reality)” (*ibid*, 291). Hintikka’s objection to this interpretation is not so much that it is false, but that it is “seriously oversimplified” (*ibid*). The central issue here turns on the fact that, when we receive a signal *s* encoded in terms of a full first-order language, we cannot always tell which of the alternatives *s* admits of are genuine as opposed to merely apparent. The non-trivially inconsistent constituents in the expansion of *s* represent these ‘apparent’ alternatives. Hintikka says that the ‘apparent’ alternatives themselves “have nothing to do with external reality but are, rather, contributed by the conceptual system we use to express our knowledge” (*ibid*, 291-2). A better understanding of the consequences of our conceptual system just *is* to increase the amount of surface information we possess; hence it is to increase the number of apparent alternatives that we can rule out (*ibid*). This understanding of our conceptual system will thus give us a refined understanding of what information *s* carries about the external world (given that *s* is true of course). It is in precisely this sense that increasing our understanding of (gaining information about) our conceptual system can yield to us information about “mind-independent reality” (*ibid*). Thus, according to Hintikka, signals encoded in a first-order polyadic language contain information about the world as well as information about our own concepts. These are “inextricably intertwined” owing to the failure of depth information to be effectively computable (*ibid*).

This move of Hintikka’s gives us a deeper understanding of how his disproof method is supposed to be an *auxiliary* method for arriving at a measure of the information yield of deductive inferences, irrespectively of how their actual proof was carried out. Hintikka understands his disproof method to somehow *reflect* the conceptual apparatus or cognitive procedures underlying our logical reasoning. We can rightly ask what good reason we might have *ever* to understand some particular formal process worked out with a piece of paper and a pencil to *reflect* the *actual* cognitive procedures that are active when we reason about the subject matter in question. Be the subject matter a proof in formal logic or not, such a move requires considerable argumentation even to begin to be persuasive. As for Hintikka’s particular version of such a move, we certainly have good reason to reject *it*. We can reject it on the grounds that it simply leaves out too many deductions for it to be a plausible reflection of the conceptual apparatus that underlies our formal reasoning. This fact is brought out by examples such as (6.5) and (6.6) above. There does not seem to be any way in which we can make a distinction *in terms of* our *underlying* conceptual apparatus between such examples and those that receive a non-zero measure of surface information.

The relationship between our conceptual apparatus and the meaning or informativeness of linguistic terms is, to understate, a vexed issue. Hintikka has something to say about the relationship between linguistic meaning in general and his distinction between depth information and surface information, however it borders on a *reductio*.

In his (1970, 292) Hintikka states that the linguistic meaning of a sentence cannot be identified with the depth information it carries, as linguistic meaning has to be something that anyone who knows the language in question can find out. Presumably he is taking the undecidability of the polyadic predicate calculus to rule out the identification of linguistic meaning with depth information for any sentence. Let us grant Hintikka this. He then suggests that linguistic meaning be related to surface information. It's difficult to know how far he intends this idea to be pushed, but the objections to it are immediate and obvious. If surface information is linguistic meaning then all logical truths of the propositional calculus, monadic predicate calculus, and some of the logical truths of the polyadic predicate calculus will fail to have any linguistic meaning due to their yielding a zero measure of surface information. Not only is this obviously false, it seems incompatible with Hintikka's own stance against the positivist position on the informativeness of logical truths. Although we may wish to say that certain logical truths are uninformative in some sense, surely we do not wish to say that they are *meaningless*, not in any sense. Furthermore, if surface information is linguistic meaning then logically equivalent sentences of the propositional calculus, monadic predicate calculus, and some of the logical equivalences of the polyadic predicate calculus will be *synonymous*. Consider an instantiation of $A \leftrightarrow ((A \wedge B) \vee (A \wedge \neg B))$ we have (6.7).

(6.7) *Grass is green* if and only if *grass is green and Tony Blair is the Prime Minister or grass is green and Tony Blair is not the Prime Minister*.

If surface information is linguistic meaning then the left hand side of (6.7) is *synonymous* with the right hand side. This too is false.

The problem of obtaining a measure of the information yield of deductive inferences is, as should now be clear, a special instance of the more general problem of obtaining a measure of the information yield of logical truths. The problem of obtaining a measure of the information yield of logical truths is, in turn, a special instance of the more general problem of obtaining *distinct* measures of the information yield of logical equivalences. This is a central issue in philosophical logic and the philosophy of language, going back at least to Frege and the functional requirements he imposed on his notion of *sense*. Hintikka's attempt to obtain a measure of the information yield of logical inferences is rightly understood as a fragment of this larger issue. It was the wish for a well-defined substitute for Frege's arguably badly defined notion that motivated Carnap to formulate his modally based notion of an *intension*, and we know from §1 (in particular (1.1)) above the relationship obtaining between intensions and the constitutive elements of *CSI*. It was the dissatisfaction with *CSI*'s treatment of logical truths, and hence of deductive inferences, that motivated Hintikka to attempt to improve this treatment via the disproof method, and the distinction between surface information and depth information. Now that this attempt has been seen to fail, what should we take away? We should abandon any attempt to obtain a measure of the information yield of logical truths, and hence of deductive inferences, via reading information off of syntax. Such syntactic approaches to the problem should be rejected in favour of semantic approaches.[10]

[10] For such an approach at the more general level of logical equivalence, see Sequoiah-Grayson's (2006).

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